'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2023

CC4-MATHEMATICS

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

Answer any *four* questions from the following $3\times4=12$

1. Solve $(x+y+1)\dfrac{dy}{dx}=1$. 3

2. If $\vec{a},\ \vec{b},\ \vec{c}$ be three vectors, show that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}]=[\vec{a}\ \vec{b}\ \vec{c}]^2$. 3

3. State Lipschitz condition. Show that the function $f(x,y)=xy^2$ satisfies Lipschitz condition on the region $|x|\leq 1,\ |y|\leq 1$. 3

4. Show that the differential equation $x^3\dfrac{d^3y}{dx^3}-6x\dfrac{dy}{dx}+12y=0$ has three linearly independent solutions of the form $y=x^r$. 3

5. A force $3\hat{i}+\hat{k}$ acts through the point $2\hat{i}-\hat{j}+3\hat{k}$. Find the torque of the force about the point $\hat{i}+2\hat{j}-\hat{k}$. 3

6. Find the linear differential equation with real constant coefficient that is satisfied by the function. $y=9-3x+\dfrac{1}{6}e^{5x}$. 3

GROUP-B

Answer any *four* questions from the following $6\times4=24$

7. Solve by the method of variation of parameters: 6

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}.$$

8. Let $\vec{F}(t) = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$. Find $\int_1^2 \vec{F}(t) \times \dfrac{d^2\vec{F}(t)}{dt^2}\, dt$. 6

9. Solve the following differential equation $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = \log x \sin(\log x)$ 6

10. If $\vec{r} = (a\cos t)\,\hat{i} + (a\sin t)\,\hat{j} + (at\tan\alpha)\,\hat{k}$, then prove that 6

$$\left[\frac{d\vec{r}}{dt},\ \frac{d^2\vec{r}}{dt^2},\ \frac{d^3\vec{r}}{dt^3}\right] = a^3\tan\alpha$$

11. Solve the equation $(D^2 - 2D + 1)y = xe^x$, by the method of undetermined coefficients. 6

12. Solve : $\dfrac{dx}{dt} - 7x + y = 0$ 6

$\dfrac{dy}{dt} - 2x - 5y = 0$

GROUP-C

Answer any *two* questions of the following 12×2 = 24

13.(a) Show that the equation of the curve, whose slope at any point (x, y) is equal to $y + 2x$ and which passes through the origin, is $y = 2(e^x - x - 1)$ 6

(b) Show that the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. Find the scalar potential u such that $\vec{F} = \text{grad}\, u$. 6

14.(a) Solve $x^2\dfrac{d^2y}{dx^2} - x(x+2)\dfrac{dy}{dx} + (x+2)y = x^3$ given that $y = x$ and $y = xe^x$ are two linearly independent solutions of the corresponding homogeneous system. 6

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ prove that $\text{curl}\,(f(r)\,\vec{r}) = \vec{0}$. 6

15.(a) Solve $\dfrac{d^2y}{dx^2} - \dfrac{2}{x}\dfrac{dy}{dx} + \left(a^2 + \dfrac{2}{x^2}\right)y = 0$ by reducing to normal form. 6

(b) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that $(\text{grad}\, u)\cdot\{(\text{grad}\, v)\times(\text{grad}\, w)\} = 0$. 6

16.(a) Prove that $(x + y + 1)^{-4}$ is an integrating factor of the equation $(2xy - y^2 - y)\,dx + (2xy - x^2 - x)\,dy = 0$ and hence solve it. 7

(b) If $\vec{F} = zy\hat{i} + z\hat{j} + y^2x\hat{k}$, where C is the curve: $x^2 + y^2 = 1, z = 0$, then find the value of $\oint_C \vec{F}\cdot d\vec{r}$. 5

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DIFFERENTIAL EQUATION AND VECTOR CALCULUS

(OLD SYLLABUS 2018)

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GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Find $\frac{1}{D^2-9}\{e^{3x}\cosh x + e^{3x}\cdot x^2 - \sin x\}$.

2. If $\vec{a},\ \vec{b},\ \vec{c}$ be three vectors, show that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$.

3. State Lipschitz condition. Show that the function $f(x, y) = xy^2$ satisfies Lipschitz condition on the region $|x|\le 1,\ |y|\le 1$.

4. Show that the differential equation $x^3\frac{d^3y}{dx^3} - 6x\frac{dy}{dx} + 12y = 0$ has three linearly independent solutions of the form $y = x^r$.

5. A force $3\hat{i}+\hat{k}$ acts through the point $2\hat{i}-\hat{j}+3\hat{k}$. Find the torque of the force about the point $\hat{i}+2\hat{j}-\hat{k}$.

6. Locate and classify the singular points of the equation
$$x^2(x^2-4)\frac{d^2y}{dx^2} + 3x^3\frac{dy}{dx} + 4y = 0.$$

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$.

8. Let $\vec{F}(t) = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$. Find $\int_1^2 \vec{F}(t) \times \frac{d^2\vec{F}(t)}{dt^2}\, dt$.

9. Solve the following differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \cdot \sin(\log x)$.

10. If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + at\tan\alpha\,\hat{k}$, then prove that

$$\left[\frac{d\vec{r}}{dt},\ \frac{d^2\vec{r}}{dt^2},\ \frac{d^3\vec{r}}{dt^3}\right] = a^3\tan\alpha$$

11. Solve the equation $(D^2 - 2D + 1)y = xe^x$ by the method of undetermined coefficients.

12. Solve : $\frac{dx}{dt} - 7x + y = 0$

$\frac{dy}{dt} - 2x - 5y = 0$

GROUP-C

Answer any *two* questions of the following 12×2 = 24

13. (a) Find the power series solution of $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0$ about the point $x = 0$. 6

(b) Show that the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. Find the scalar potential u such that $\vec{F} = \text{grad}\,u$. 6

14.(a) Solve $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = x^3$ given that $y = x$ and $y = xe^x$ are two linearly independent solutions of the corresponding homogeneous system. 6

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}\,|$, prove that $\text{curl}\,(f(r)\,\vec{r}) = \vec{0}$. 6

15.(a) Solve $\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(a^2 + \frac{2}{x^2}\right)y = 0$ by reducing to normal form. 6

(b) If $u = x + y + z,\ v = x^2 + y^2 + z^2,\ w = yz + zx + xy,$ prove that $(\text{grad}\,u)\cdot\{(\text{grad}\,v)\times(\text{grad}\,w)\} = 0$. 6

16.(a) Solve $(D^3 - D^2 + 3D + 5)y = e^x\cos 3x$. 6

(b) If $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$, where C is the curve: $x^2 + y^2 = 1,\ z = 0$, then find the value of $\oint_C \vec{F}\cdot d\vec{r}$. 6

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